Universal deblurring method for real images using transition region

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Abstract. In this paper, we present a universal deblurring method for real images without prior knowledge of the blur source. The proposed method uses the transition region of the blurred image to estimate the point spread function (PSF). It determines the main edges of the blurred image with high edge measures based on the difference of Gaussians (DoG) operator. Those edge measures are used to predict the transition region of the sharp image. By using the transition region, we select the pixels of the blurred image to form a series of equations for calculating the PSF. In order to overcome noise disturbance, the optimal method based on the anisotropic adaptive regularization is used to estimate the PSF, in which the constraints of non-negative and spatial correlations are incorporated. Once the PSF is estimated, the blurred image is effectively recovered by employing nonblind restoration. Experimental results show that the proposed method performs effectively for real images with different blur sources. © 2012 Society of Photo-Optical Instrumentation Engineers (SPIE). [DOI: 10.1117/1.OE.51.4.047006]

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1 Introduction

Image blur is caused by different factors in different situations. The existing deblurring methods focus on remedying deblurred images with known blur sources, such as motion, defocus, turbulence, and so on. When the blur sources are known a priori, the conventional methods usually attempt to estimate point spread function (PSF) using analytical models. Note that one-dimensional (1-D) uniform model is used for horizontal motion blur. Often, the blur caused by long-term exposure through the atmosphere is modeled by a Gaussian PSF.

When a blur source is identified as object motion, the existing deblurring methods obtain the PSF models from motion-blurred images. Yitzhaky and Kopeika identify PSFs from the motion-blurred image in the cepstrum domain. Sawchuk proposed a deblurring method by analyzing the space variant system of image motion. Sawchuk’s method gives a mechanical description of motion and subsequently derives an equivalent linear space variant system containing all the motion effects. Stern et al. proposed a restoration and resolution-enhancement method using a vibration-distorted image sequence. In order to obtain the motion parameters, a few algorithms have been proposed to identify periodic zeros in the power spectrum of the uniform motion blurred images. However, it is not trivial to find spectral zeros in noisy conditions. Fergus et al. proposed a variation of Bayesian approach using the statistical property of the image gradient distribution to approximate the unblurred image when the motion parameters are unknown. Shan et al. proposed an effective high-quality motion-deblurring method based on a probability distribution model of a single image. Fergus’s and Shan’s methods mainly use the prior probability on the edge distribution of images. All the above mentioned deblurring methods use a priori information of motion to restore blurred images. They work well on motion-blurred images, but they are unsuitable for images blurred by other sources. Additionally, these deblurring methods cannot work in cases when the blur sources are unknown.

For images blurred by defocus and atmospheric turbulence, conventional methods also attempt to obtain the PSF models from blur sources. Defocus blur is modeled as a uniform intensity distribution within a circular disk. Tai et al. proposed a deblurring method for a single image with defocus blur. For the turbulence-degraded images, Frieden et al. proposed a method that models the PSF of the turbulence as a process of stochastic superposition of Gaussian speckles. Wang et al. proposed a restoration method based on the multicriteria neural network approach to solve the weights and displacements of speckles. Note that Gaussian PSF is used in Frieden’s and Wang’s methods. In reality, a complete model of blur operator depends on a few parameters. Accurate knowledge of all these parameters...
is often unavailable after the image has been taken. Furthermore, it is difficult to judge blur sources or blur modes based on the observed images. In real situations, the blur of the captured image might be caused by more than one source. For example, the motion blur of real images usually comes together with defocus blur. Therefore, it is necessary to propose a universal deblurring method for real images in which the blur modes do not need to be identified before performing image restoration. In addition, conventional blind deblurring methods usually use the whole image region to estimate the PSF in their alternating iterations, resulting in inaccurate and time-consuming PSF estimation. Furthermore, most of the existing blind deblurring methods work well only on synthetic cases but fail on real data.

We observe that blur information is mainly retained in the transition region (the pixels between the object and background) in the blurred image. In fact, the smooth area of the object and background does not help estimate the PSF. Unlike the existing methods that use the whole image to estimate PSF, the proposed method uses the transition region of the blurred image. Different blurred images have different blur models. However, their PSFs are finite in their two-dimensional (2-D) support. If the discrete values of the overall PSF can be estimated by 2-D support, it is possible to develop a universal deblurring method for the images blurred by different sources.

In this paper, we estimate PSF using the transition region of the blurred image. The transition region is predicted by locating the edges of blurred images, while the location of edges is determined by using the difference of Gaussians (DoG) operator. After the edges of the deblurred image are located correctly, we predict transition regions of the sharp image, which are around the predicted edges. Once the transition region of the sharp image is obtained, the PSF is estimated by an optimal method based on anisotropic adaptive regularization, in which the constraints of non-negative and spatial correlations are incorporated. By obtaining the PSF, we employ the improved maximum likelihood (ML) estimation deblurring method to recover the blurred images.

This paper is organized as follows. Section 2 discusses edge detection with edge strength measurement. Section 3 describes the prediction of the transition region of a sharp image. The optimal PSF estimation is described subsequently in Sec. 4. The experimental results are presented in Sec. 5. Finally, we give conclusive remarks in Sec. 6.

2 Edge Detection with Edge Strength Measurement

Obviously, edge detection on a blurred image is much more difficult than that on an unblurred one. Existing edge detection methods do not include any measurement for adjusting parameters for edge strength, which results in many weak and false edges. These weak and false edges affect accuracy of PSF estimation. For example, Huertas et al. proposed an effective edge detection method by detecting zero-crossing using a Laplacian of Gaussian (LoG) operator. However, it still has the shortcomings mentioned above.

In order to overcome the problem, we use a DoG operator instead of LoG and propose the concept of edge measurement to extract the proper edges. 1-D and 2-D DoG functions are shown in Fig. 1, in which the 2-D DoG function is expressed by:

\[
\text{DoG}(x, y) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{x^2+y^2}{2\sigma_1^2}\right) - \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{x^2+y^2}{2\sigma_2^2}\right). \tag{1}
\]

Note that since DoG is a linear operator, applying DoG to an image is equivalent to the subtraction of the Gaussian filtered images with different variances.

When we convolve the DoG operator with a sharp edge and a corresponding blurred edge, respectively, the detected edge is both located at the zero-crossing [Fig. 2(a)–2(c)]. Consequently, we can extract the edge of a blurred image by seeking the zero-crossing points. In order to extract desired edges from these zero-crossing points, we propose an edge strength metric that is evaluated by distance \(p\) between the peak and valley around the zero-crossing points as shown in Fig. 2(d) and 2(e). Then we can choose the main edges with a large value to avoid false and weak edges.

3 Prediction of Transition Region of Sharp Image

When images are blurred, the edge in blurred images is closely corresponding to those in sharp images. On the other hand, the distributions of pixel values on both sides of the edges are different. Therefore, useful information of the transition region of the sharp image can be predicted when edges are located.

The range of the transition region of a sharp image can be predicted by finding the upper- and lower-limit pixel values on both sides of edges (Fig. 3). To find the upper-limit

![Fig. 1 DoG operator. (a) One-dimensional DoG; (b) Two-dimensional DoG.](image-url)
values, we search the increasing pixel values along the orthogonal direction of the edge line. When the upper-limit values are found, we define the region between the detected edge and pixels with the upper-limit values as the upper limit transition band. Similarly, we can find the lower-limit values of pixels on the other side of the edges and define the corresponding lower-limit transition band. The range of the transition region of a sharp image consists of both transition bands as shown in Fig. 3.

When the range of the transition region has been found, we need to predict the pixel values in the transition region. The pixel value on the edges is set to the weighted average of the upper- and lower-limit values. Note that the weight is determined by the width of each transition band. Furthermore, the pixel values in the upper- and lower-limit bands are set to the pixel values at the corresponding upper and lower limits, respectively. When the edge line is oblique, the pixel information on both sides of the edge line can retain continuity by using the bilinear interpolation method. Then we obtain the pixel values of the transition region of the sharp image.

After the transition region of the sharp image is predicted, we can then estimate the PSF. Note that only the pixels within a certain radius of the predicted edges are used to estimate the PSF, since the pixels away from the transition region do not carry accurate information.

4 Optimal PSF Estimation
The simple image blur model without noise is described as

\[ g(i, j) = (f \ast h)(i, j), \]  

where \( f \) is the latent image and \( h \) is the blur PSF in \( M \times M \) support, respectively. Equation (2) can be expressed in the matrix vector form as follows:

\[
\begin{bmatrix}
h(0, 0) \\
\vdots \\
h(x, y) \\
\vdots \\
h(M-1, M-1)
\end{bmatrix}
\begin{bmatrix}
f(i-x, j-y), \ldots \end{bmatrix}_{1 \times M^2}
= g(i, j),
\]  

where \( x = 0, 1, \ldots, M-1 \) and \( y = 0, 1, \ldots, M-1 \).

From Eq. (3), we know that any point in the blurred image can form an equation of PSF \( h \). Existing deblurring methods usually use the whole or selected patch of the blurred image to estimate the PSF. In this procedure, the smooth foreground and background of the blurred object are not helpful for estimating the PSF, since the problem becomes ill-conditioned. In this paper, we use only the transition region to estimate PSF. Consequently, it avoids existing problems. We select \( K \) points from the blurred image to form a linear matrix equation as given by
Ax = b, \tag{4}

where A is a matrix of size $M^2 \times K$ whose row elements come from the pixels in a square area in the transition region $f$ [see Eq. (3)], $x$ is a column vector of PSF whose elements are stacked in a row-major order, and $b$ is a column vector whose elements are the observed pixel values in the blurred image. In order to estimate the PSF more robustly, the number of points in the support of PSF is selected, whose elements are the observed pixel values in the blurred image. In order to easily solve the PSF, we employ the lagged diffusivity fixed-point (FP) iteration method\(^a\) to determine $\alpha(|\nabla x(i,k)|)$. According to the FP method, $\alpha(|\nabla x(i,k)|)$ is determined by the known values of the last step ($n-1$) in iterations.

When $\alpha(|\nabla x(i,k)|)$ is fixed, we minimize the criterion function in Eq. (6) by setting its derivative to zero, which can be written as

$$\hat{x}^{(n)} = \arg\min_x \{J^{(n)}(x^{(n-1)}, \alpha(|\nabla x^{(n-1)}|))\} \tag{5}$$

$$\Rightarrow \frac{\partial J^{(n)}}{\partial x^{(n)}}|_{x^{(n−1)}, \hat{x}^{(n)}} = 0. \tag{6}$$

$$\hat{x}^{(n)} = \frac{A^T b}{A^T \Lambda + \phi_1 \Lambda^{(n-1)} + \phi_2 D^{(n-1)}} , \tag{7}$$

until the stopping criterion $\|\hat{x}^{(n)} - \hat{x}^{(n-1)}\| \leq \varepsilon$ is satisfied, or the iteration count reaches the given maximum iterative number.

When PSF $x$ is obtained, the blurred image can be recovered by using existing nonblind deblurring methods. In order to preserve the edges and suppress the artifacts in the smoothing areas, in this paper, we use the improved maximum likelihood estimation deblurring algorithm (ML)\(^4\) to recover the blurred image, which can be described by

\[
\hat{f}^{(n+1)}(x) = \frac{1}{1 + \eta L(\nabla f^{(n)})} \times \sum_{y \in Y} \frac{h(y - x)}{\sum_{z \in X} h(y - z) f^{(n)}(z)}, \tag{8}
\]

where $\eta$ is small constant coefficient and $L(\nabla f^{(n)})$ is anisotropic regularization term\(^4\) to preserve edges while smoothing noise during iterative restoration.

### 5 Experimental Results

In order to verify the effectiveness and the stability of the proposed method, we perform a series of deblurring experiments on real different spectral images caused by different sources. In our implement, 2-D DoG in Eq. (1) turns out to be a separable operator. Therefore, we use 1-D convolution in a vertical and
horizontal direction independently instead of applying 2-D convolution directly, which reduces the computational burden. The experiments are carried out on a 2.67-GHz CPU with 4-GB RAM. The algorithm is implemented with C language in VC6.0 environment.

To evaluate the proposed method quantitatively, we begin with a synthetic example with known PSF and image. Figure 4(b) is a synthetic turbulence-degraded image, which is generated by convolving the original image in Fig. 4(a) with Gaussian PSF (σ = 3, support size is 21 × 21). The PSF is shown in the red right-bottom window in Fig. 4(b). To show the effectiveness of the proposed method in restoring turbulence-degraded images, we compare it with Fish et al.’s method,\textsuperscript{19} the NAS-RIF method,\textsuperscript{20} and the TV method.\textsuperscript{21} In our implementation, the constants φ₁ and φ₂ in Eq. (6) are adjustable. φ₁ = 1 × 10\textsuperscript{7} and φ₂ = 4.5 are the default values in our implementation, both of which are also suitable for this experiment.

By running Fish et al.’s, NAS-RIF, TV, and the proposed method, we obtain the deblurred images given in Fig. 4(c)–4(f), respectively. The estimated PSFs are shown in their red right-bottom windows, except the NAS-RIF method, which does not need to estimate PSF. From Fig. 4(c)–4(f), it is observed that the result of the proposed method is better than others. To measure the performance quantitatively, the peak signal-to-noise ratio (PSNR)\textsuperscript{22} is calculated, which is defined as

\[
\text{PSNR} = 10 \log_{10} \left( \frac{(\text{MAX} - \text{MIN})^2}{\text{MSE}} \right),
\]

where MSE = \(\frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} (\hat{f}(i,j) - f(i,j))^2\), MAX and MIN is the maximum and minimum possible pixel values of the image. The computed PSNRs for different deblurring methods are listed in Table 1, in which it is observed that the PSNR value of the proposed method is better than the others, which means the proposed method quantitatively outperforms the others.

Next, we validate the proposed method with a motion-blurred image. We compare the performance with the state-of-the-art motion-deblurring methods by Fergus et al.\textsuperscript{6} and Shan et al.\textsuperscript{7} Figure 5 is a portion of a real motion-blurred image of a running bus on a sloped road, with an image size of 320 × 230. The patch indicated in Fig. 5 in the red rectangle is used to estimate PSF in Fergus et al.’s method. Since the methods of Fergus et al. and Shan et al. require prior knowledge of kernel size, we set the kernel size to 21 × 21 pixels. On the other hand, the proposed method automatically produces the kernel size by determining the distance between the zero-crossing point and the peak point (see Fig. 2). However, for a fair comparison of running time, we also set the kernel size to 21 × 21. Running Fergus et al.’s method (using their Matlab codes), Shan et al.’s method (using their executable codes), and the proposed method, we get the estimated PSFs shown in Fig. 6(a), 6(c), and 6(e), respectively. It is obvious that the estimated PSF of the proposed method is narrow and consistent, and the visual quality is better than that of the others. After the PSFs are estimated, for fair comparison, we apply the ML algorithm\textsuperscript{14} to perform deblurring with the same iteration number. In this experiment, we use the default iteration numbers 3 and 60 for the iteration numbers of the outer loops and inner loops, respectively. The deblurred results are given in Fig. 6(b), 6(d), and 6(f), which also show that the proposed method subjectively achieves better deblurring. In addition, in order to compare the computational complexity of all methods, we list the runtime of the methods in Table 2, which indicates that the proposed method spends much less time than the others.

Next, we test the efficiency of the proposed method on a blurred image by camera defocus. Figure 7(a) is a defocused image that is captured by a Canon EOS 350D. Figure 7(b) is the deblurred result using the proposed method. Obviously, the defocus blur in Fig. 7(a) is removed efficiently while exhibiting richer and clearer details.

We verify the deblurring result of the proposed method for a turbulence-degraded image. Figure 8(a) is a turbulence-degraded image taken in a wind-tunnel test. Using our method, we get the deblurred image shown in Fig. 8(b).

### Table 1: Comparison of PSNR for the deblurred images.

<table>
<thead>
<tr>
<th></th>
<th>Blurred</th>
<th>Fish et al.’s</th>
<th>NAS-RIF</th>
<th>TV</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSNR</td>
<td>16.1717</td>
<td>16.4877</td>
<td>17.5977</td>
<td>18.8410</td>
<td>19.3188</td>
</tr>
</tbody>
</table>

Fig. 4 Comparison of restoration: (a) original image; (b) a synthetic turbulence-degraded image; (c) deblurred by Fish et al.’s method; (d) deblurred by the NAS-RIF method; (e) deblurred by the TV method; (f) deblurred by the proposed method.

Fig. 5 Motion blurred image. The patch indicated by the red rectangle is used to estimate the PSF in Fergus et al.’s method.
On the contrary, Fig. 8(c) is the deblurred image using the TV method, which shows poorer performance both estimating the PSF and deblurring.

To validate the universality of the proposed method, we test it on a blurred image caused by more than one source. Figure 9(a) is a blurred image of the end of a moving steel billet in hot rolling production line, which is blurred by the combination of defocus and motion. We use the proposed method to recover it and get the deblurred result shown in Fig. 9(b). The deblurred result shows that the proposed method is also effective on blurred images caused by more than one source.

![Fig. 8](image)

![Fig. 9](image)

Table 2 Comparison of runtime.

<table>
<thead>
<tr>
<th>Method</th>
<th>Fergus et al.’s</th>
<th>Shan et al.’s</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Running time (sec)</td>
<td>183</td>
<td>97</td>
<td>45</td>
</tr>
</tbody>
</table>

Fig. 7 Deblurring of defocus blurred image: (a) defocus image; (b) deblurred by the proposed method.

We further test the proposed method for a set of blurred infrared images and compare the results to those of Fish et al.’s and TV method. Given a blurred infrared
image shown in Fig. 10(a). Figure 10(b)–10(d) is the deblurred results using Fish et al.’s, TV, and the proposed methods, respectively. The runtime of these methods is listed in Table 3, which shows that our method consumes significantly less time. By comparing the deblurred results shown in Fig. 10(b)–10(d), we can see that the result of using the proposed method is clearer and has a better visual effect. Figure 11(a) is another infrared image taken by a forward looking infrared (FLIR) T330 camera. Using the proposed method, we get the deblurred image shown in Fig. 11(b). Obviously, the blur is effectively alleviated.

Finally, we test the proposed method with a millimeter-wave blurred image and a terahertz (THz) blurred image. Figure 12(a) is a millimeter-wave blurred image. Figure 12(b) is the deblurred result using the proposed method, which shows that the proposed method is still effective for a millimeter-wave blurred image. Figure 13(a) is a THz blurred image.

<table>
<thead>
<tr>
<th>Method</th>
<th>Fish et al.’s</th>
<th>TV</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Running time (sec)</td>
<td>21.5</td>
<td>7.8</td>
<td>5.1</td>
</tr>
</tbody>
</table>
which is a handbag with a cutter, an optical disk, and a pen inside. Figure 13(b) is the deblurred result using the proposed method. Obviously, the real THz image is significantly deblurred using the proposed method.

6 Conclusions
In this paper, we proposed a universal deblurring method for real images without prior knowledge of the blur source. The transition region of the sharp image is extracted to estimate the point spread function (PSF) correctly, based on the difference of Gaussians (DoG) operator. Those edge measures are used to predict the transition region of the sharp image. The optimal method based on the anisotropic adaptive regularization is used to estimate the PSF, and the blurred image is recovered using an existing nonblind restoration method. The experimental result showed that the blur caused by different blur sources, such as motion, defocus, and turbulence, is removed efficiently without adjusting the algorithm to particular blur sources.

Appendix: Derivation of Eq. (14)
In this Appendix, we give the derivation of Eq. (14). From Eq. (13), we have the following derivation:
\begin{align}
\frac{\partial J(x^{(n)})}{\partial x^{(n)}} &= \frac{\partial (\|Ax^{(n)} - b\|^2)}{\partial x^{(n)}} + \frac{\partial (q_1 \|A^{(n-1)}x^{(n)}\|^2)}{\partial x^{(n)}} \\
&= \frac{\partial \left\{ q_2 \sum_{i} \alpha(\|\nabla x^{(n-1)}(i,k)\|) \left(x^{(n)}_i - x^{(n)}_j\right)^2 \right\}}{\partial x^{(n)}} \\
&+ 2A^T(Ax^{(n)} - b) + 2q_1A^{(n-1)}x^{(n)} \\
&+ 2q_2 \sum_{i} \alpha(\|\nabla x^{(n-1)}(i,k)\|) \left(x^{(n)}_i - x^{(n)}_j\right)^2.
\end{align}

(17)

According to the FP method mentioned in Sec. 4, \( \alpha(\|\nabla x^{(n-1)}(i,k)\|) \) is known by the values of the last iteration \((n - 1)\). The third term can be expressed by \( DX \), where \( D \) is the matrix of the anisotropic coefficients. Then Eq. (17) can be written as

\begin{align}
\frac{\partial J(x^{(n)})}{\partial x^{(n)}} &= 2A^T[Ax^{(n)} - b] + 2q_1A^{(n-1)}x^{(n)} + 2q_2D^{(n-1)}x^{(n)}.
\end{align}

(18)

Setting Eq. (18) to zero to minimize the criterion function in Eq. (7), we can get Eq. (14) by

\begin{equation}
A^T[Ax^{(n)} - b] + 2q_1A^{(n-1)}x^{(n)} + 2q_2D^{(n-1)}x^{(n)} = 0.
\end{equation}

We take the current point \( x_t \) for example, to construct matrix \( D \). Suppose that the 2-D coordinate of \( x_t \) is \((s, t)\), then we have \( i = sM + t \). For the sake of a concise expression, we mark \( \alpha_m, x_{s,t}, x_k \) to replace \( \alpha(\|\nabla x^{(n-1)}(i,k)\|) \), \( x^{(n)}_i \), \( x^{(n)}_k \), respectively. \( m \in \{1, 2, 3, 4, 5, 6, 7, 8\} \), \( x_k \in \{x_{s-1,t-1}, x_{s-1,t-1}, x_{s-1,t-1}, x_{s-1,t-1}, x_{s-1,t-1}, x_{s-1,t-1}, x_{s-1,t-1}, x_{s-1,t-1}\} \).

The \( i \)th part of the third term in Eq. (17) can be expressed by

\begin{equation}
\alpha \left\{ \sum_{k} \alpha(\|\nabla x^{(n-1)}(i,k)\|) \left(x^{(n)}_k - x^{(n)}_j\right)^2 \right\}
\end{equation}

\begin{equation}
\frac{\partial}{\partial x^{(n)}} = 2\sum_{k} \alpha(\|\nabla x^{(n-1)}(i,k)\|) \left(x^{(n)}_k - x^{(n)}_j\right)^2 = 2\sum_{m} \alpha_m(x_k - x_i)
\end{equation}

\begin{equation}
= 2\alpha_1(x_{s-1,t-1} - x_{s,t}) + 2\alpha_2(x_{s-1,t-1} - x_{s,t}) \\
+ 2\alpha_3(x_{s-1,t-1} - x_{s,t}) + 2\alpha_4(x_{s-1,t-1} - x_{s,t}) \\
+ 2\alpha_5(x_{s-1,t-1} - x_{s,t}) + 2\alpha_6(x_{s-1,t-1} - x_{s,t}) \\
+ 2\alpha_7(x_{s+1,t-1} - x_{s,t}) + 2\alpha_8(x_{s+1,t-1} - x_{s,t}) \\
= 2(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8) x_{s,t}.
\end{equation}

(19)

The coefficients of each term in Eq. (19) are the elements of the \( i \)th row of matrix \( D \), with a total of nine elements. The other elements, except the nine elements in the \( i \)th row, are zeros. The positions of column \( f_k \) (\( k = 1, 2, \ldots, 9 \)) of the nine elements are determined by the 2-D coordinates of \( x \). Now we have

\begin{equation}
\begin{array}{l}
\alpha_1 = M + (t - 1), \quad \alpha_2 = (s - 1)M + t, \\
\alpha_3 = (s - 1)M + (t + 1), \\
\alpha_4 = sM + (t - 1), \\
\alpha_5 = sM + t, \quad \alpha_6 = sM + (t + 1), \\
\alpha_7 = (s + 1)M + (t - 1), \quad \alpha_8 = (s + 1)M + t, \\
\alpha_9 = (s + 1)M + (t + 1),
\end{array}
\end{equation}

Similarly, we can construct the 0, 1, \ldots, \( M^2 - 1 \)th row of matrix \( D \) from all the points \( x_0, x_1, \ldots, x_{M^2-1} \), respectively. Then we obtain matrix \( D \) as follows:

\begin{equation}
D = \begin{bmatrix}
0, 1, \ldots, j_1, j_2, j_3, \ldots, j_4, j_5, j_6, \ldots, j_7, j_8, j_9, \ldots, M^2 - 1
\end{bmatrix}
\end{equation}

\begin{equation}
\begin{bmatrix}
(0, 1, \ldots, M^2 - 1)
\end{bmatrix}
\end{equation}

(20)

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References


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